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MONITORING DECLINING QUALITY OF AMMUNITION STOCKPILE UNDER STEP-STRESS

So Young Sohn

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Monitoring Declining Quality of Ammunition Stockpile under Step-Stress

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Abstract

Most ammunition is produced long before its ultimate consumption and stored in a series of different depots for a considerably long period of time. During storage, the quality of the ammunition stockpile deteriorates proportionally to the conditions of depots. We view different conditions associated with a series of depots as step-stress. A random effects logistic regression model is employed to predict the quality of ammunition stockpile in terms of the routing information such as a series of location and duration of storage of ammunition lots. The resultant prediction model can be used to determine the appropriate time for reorder or renovation of ammunition before the quality reaches substandard. An example is given to illustrate the implementation procedure of the prediction model suggested in this paper.

Key Words: Random Effects Model, Step-Stress, Deterioration Rate, A Two-Stage Estimation

1. INTRODUCTION

Designing the proper surveillance program for the material whose quality deteriorates during storage has been one of the important topics in the area of material management (Valdez-Flores and Feldman [13], Whitehead [14]). Especially, when the degradation of the quality would cause not only economic loss but also catastrophic disaster such as loss of human life, importance of the appropriate quality control cannot be overemphasized. A good example would be ammunition lots that are stored in depots for a relatively long period of time before their ultimate usage. In order to keep ammunition stockpile from reaching substandard faster than expected, well planned surveillance programs are necessary.

In an attempt to provide inputs to such surveillance programs Sohn [10] formulated an ammunition stockpile deterioration model based on a random effect logistic regression analysis. The suggested model enables one to predict the deterioration rate of ammunition lots in terms of depot condition along with other related characteristics such as vendor sources and the manufacturing year. In order to model deteriorating patterns, one of the assumptions employed in Sohn [10] was that once ammunition lots are sent to a depot, they remain in the same depot during the experimental period.

In practice, however, ammunition lots may be transferred to several depots in sequence before ultimate usage. A typical logistics chain of ammunition lots described in Brzuskiewicz and Morrison [1] begins with the load plant. At the load plant, ammunition lots sometimes spend up to one year in temporary storage before deployment to the permanent storage area. The ammunition lots shipped to and stored in permanent depots are

then rotated from war reserve and tested on schedules which depend on the availability of the item and the policy of the Defense Ammunition Director.

In sum, an ammunition lot would be exposed to a series of different conditions of depots associated with different levels of average temperature and humidity in various locations (underground depot, aboveground depot, and warship)(see Eriksen and Strømsœ[2], Forsyth et al. [3]). A series of different conditions can be viewed as step-stress consisting of several levels of constant stress.

In this paper, we formulate the deteriorating pattern of the quality of ammunition stockpile under step-stress in order to accommodate the possible exposure of ammunition lots to several different depots. The main goal is to provide tools to predict the quality of ammunition given the expected duration and locations of storage of ammunition lots in sequence. In section 2, the model formulation (a random effects logistic regression) under constant stress and the necessary estimation methods introduced in (Sohn [10]) is briefly reviewed. In section 3, by combining the segments of individual models, a prediction model for ammunition deterioration under step-stress is derived. In section 4, an example is given to illustrate the implementation procedure. Finally, discussion is given in section 5.

2. CONSTANT STRESS MODEL

Consider the following experiment. N lots of homogeneous caliber ammunition (say, fuze manufactured by the same vendor in the same year) are purchased from a vendor. They are sent to m different depots: lot number $1, .., N_1$ to depot 1; lot number $N_1+1, .., N_1+N_2$ to depot 2; ,.., ; lot number $N_{m-1}+1, .., N_{m-1}+N_m = N$ to depot m.

It is assumed that once lots are stored in a depot, they remain in the

same depot during the experimental period. As a result of acceptance sampling, the qualities of incoming ammunition lots are assumed to be homogeneous while the average deterioration rates of ammunition lots may vary depending upon conditions of depot in which ammunition lots are stored. Although environmental conditions associated with a depot vary continually over time, the average condition of a depot is considered as constant stress given to ammunition lots in the depot.

In order to estimate deteriorating patterns of ammunition lots over time under constant stress, each lot i ($i = 1,.,N$) is repeatedly inspected on a sampling basis without rectification. The number of defective items (y_{ij}) found out of sample size (n_{ij}) at the jth inspection of lot i ($j = 1,..,n_i$) would follow a binomial distribution with a parameter, expected cumulative proportion defective (p_{ij}). The expected cumulative proportion defective (p_{ij}) would be a non-decreasing function of time (t_{ij}) and we use the following within-lot logistic model to describe the deteriorating pattern of lot i:

For $i = 1,..,N$, and $j = 1,..,n_i$

$$p_{ij} = exp(\beta_0 + \beta_i t_{ij})/(1 + exp(\beta_0 + \beta_i t_{ij})) \tag{1}$$

where $exp(\beta_0)/(1 + exp(\beta_0))$ represents the initial proportion defective of ammunition lots and β_i is the deterioration rate of ammunition lot i which would be positive. The average deterioration rate of ammunition lots stored in one depot often differs from that of another depot depending upon their environmental conditions. We assume it is mainly due to different depot conditions while there is some part of variation that cannot be explained by such conditions.

One of the possible models that accommodate these points would be the following between-lot model for β_i:

$\beta_i = exp(\gamma_1 z_{i1} + .. + \gamma_m z_{im} + \epsilon_i)$ or

$$ln\beta_i = \gamma_1 z_{i1} + .. + \gamma_m z_{im} + \epsilon_i \tag{2}$$

where z_{ik} $(k = 1, .., m)$ is a dummy variable (z_{ik}=1 if a lot i is stored in depot k; otherwise 0) and ϵ_i follows independent $N(0, \sigma^2)$. The corresponding regression coefficient γ_k would imply the average log(deterioration rates) of ammunition lots stored in depot k.

Once unknown regression coefficients $(\gamma_1, .., \gamma_m)$ in (2) are estimated, they can be used as a basis to examine which depots are associated with significantly higher average deterioration rates than the others as shown in [9]. In addition, when β_0 is available, one can predict the proportion defective of an ammunition lot which would be stored in one of the depots or similar depots used in the experiment.

In order to estimate unknown parameters such as β_0 and $(\gamma_1, .., \gamma_m)$, we use a two-stage method which separates the estimation of the within-lot model (1) from that of the between-lot model (2) (Korn and Whittemore [5], Sohn [10] and Stiratelli et al. [11]).

A Two-Stage Estimation

First, in order to estimate the within-individual model (1), the following likelihood function of y_{ij} conditional on β_0, β_1,...,β_N is formulated:

$$L1 = \prod_{i=1}^{N} \prod_{j=1}^{n_i} p_{ij}^{y_{ij}} (1 - p_{ij})^{n_{ij} - y_{ij}} \tag{3}$$

where p_{ij} is as in equation (1).

In order to estimate β_0, β_1,...,β_N that maximizes (3), we differentiate $ln(L1)$ with respect to β_0, β_1,...,β_N. By solving a set of resulting normal equations, we find the maximum likelihood (ML) estimates, ($\hat{\beta}_0$, $\hat{\beta}_1$,...,$\hat{\beta}_N$). A $(N+1)\times 1$ vector $\hat{\beta}$= ($\hat{\beta}_0$, $\hat{\beta}_1$,...,$\hat{\beta}_N)'$ would follow asymptotically normal distribution with mean β = (β_0, β_1,..,$\beta_N)'$ and variance Δ where

$$\Delta = \begin{pmatrix} \tau^2_{0,0}, & \tau^2_{0,1} & \cdots & \tau^2_{0,N} \\ \tau^2_{1,0} & \tau^2_{1,1} & \cdots & \tau^2_{1,2} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \tau^2_{N,0} & \cdots & \cdots & \tau^2_{N,N} \end{pmatrix}.$$

The inverse of the negative information matrix evaluated at ($\hat{\beta}_0$, $\hat{\beta}_1$,...,$\hat{\beta}_N$), $\hat{\Delta}$ can be used to estimate the variance matrix Δ.

Once the ML estimates $\hat{\beta}_i$'s are obtained, they can replace the unobservable β_i in the between-individual model (2). This replacement, however, adds the estimation error δ_i to the equation (2):

$$\ln\hat{\beta}_i = \gamma_1 z_{i1} + ... + \gamma_m z_{im} + \epsilon_i + \delta_i \tag{4}$$

for $i = 1, .., N$.

Using matrix notation model (4) can be written as follows:

$$\ln\hat{\beta} = Z\gamma + \epsilon + \delta \tag{5}$$

where $ln\hat{\beta}$ is an $N \times 1$ vector, $(ln\hat{\beta}_1, .., ln\hat{\beta}_N)'$; Z is an $N \times m$ matrix of z_{ik}'s; the γ is an $m \times 1$ vector of γ_l's; ϵ is an $N \times 1$ vector of ϵ_i's; and δ is an $N \times 1$ vector of δ_i's.

δ is assumed to be statistically independent of ϵ and would asymptotically follow normal distribution with mean 0 and variance Ω. where

$$\Omega = \begin{pmatrix} \tau_{1,1}^2/\beta_1^2, & \tau_{1,2}^2/\beta_1\beta_2 & \dots & \tau_{1,N}^2/\beta_1\beta_N \\ \tau_{2,1}^2/\beta_1\beta_2 & \tau_{2,2}^2/\beta_2^2 & \dots & \tau_{2,N}^2/\beta_2\beta_N \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \tau_{N,1}^2/\beta_N\beta_1 & \dots & \dots & \tau_{N,N}^2/\beta_N{}^2 \end{pmatrix}.$$

Estimated Ω, $\hat{\Omega}$, can be obtained by replacing $\tau_{i,k}^2/\beta_i\beta_k$ with $\hat{\tau}_{i,k}^2/\hat{\beta}_i\hat{\beta}_k$ for $i, k = 1, .., N$. In sum, $\ln\hat{\beta} \sim N(Z\gamma, V)$ where $V = \hat{\Omega} + \sigma^2$.

Based on normal $\ln\hat{\beta}$, the restricted maximum likelihood (REML) method is used to estimate the between-individual model parameter σ^2 which satisfies the following:

$$-0.5\,trR - 0.5 ln\hat{\beta}' RR\, ln\hat{\beta} = 0 \tag{6}$$

where $R = V^{-1} - V^{-1}Z(Z'V^{-1}Z)^{-1}Z'V^{-1}$ (Searle et. al [9]). Once $\hat{\sigma}^2$ is obtained from (6), it replaces σ^2 in V and γ can be estimated as

$$\hat{\gamma} = (Z'\hat{W}Z)^{-1}(Z'\hat{W}(ln\hat{\beta})) \tag{7}$$

where $\hat{W} = (\hat{\Omega} + \hat{\sigma}^2 I_N)^{-1}$ and the estimated variance of $\hat{\gamma}$ is $(Z'\hat{W}Z)^{-1}$.

Finally when the estimates $\hat{\gamma}_l$'s in (7) replace γ_l's in (4), $ln\hat{\beta}_i$ can be predicted in terms of $z_{i1}, ..., z_{im}$:

$$\widehat{ln(\hat{\beta}_i)} = \hat{\gamma}_1 z_{i1} + \dots + \hat{\gamma}_m z_{im} \tag{8}$$

or

$$\hat{\hat{\beta}}_i = exp(\hat{\gamma}_1 z_{i1} + \dots + \hat{\gamma}_m z_{im}). \tag{9}$$

Notice the difference between the estimated individual deterioration rate $\hat{\beta}_i$ and the average group deterioration rate $\hat{\hat{\beta}}_i$ that takes into account the random effects.

In order to predict the proportion defective of a randomly selected lot i' that was not used in the experiment, one can use the following:

$$\hat{p}^*_{i'}(t) = exp(\hat{\beta}_0 + \hat{\hat{\beta}}_{i'} t)/(1 + exp(\hat{\beta}_0 + \hat{\hat{\beta}}_{i'} t)) \tag{10}$$

where $\hat{\hat{\beta}}_{i'} = exp(\hat{\gamma}_1 z_{i'1}+, ..., +\hat{\gamma}_m z_{i'm})$.

Subsequently, one can estimate the expected time $t^*_{i'}$ when the quality of ammunition lot i' reaches a predetermined level p:

$$\hat{t}^*_{i'}(p) = [ln(p/(1-p)) - \hat{\beta}_0]/\hat{\hat{\beta}}_{i'}. \tag{11}$$

3. STEP-STRESS MODEL

In order to formulate the deterioration model with step-stress, the following assumptions are made based on Nelson [7]: (1) The remaining life of ammunition depends only on the current cumulative proportion defective and the current stress associated with the depot regardless how the proportion is accumulated; (2) If held at the current stress, defective items will occur according to the logistic function for that stress but starting at the previously accumulated fraction failed; (3) The change in stress has no effect on life - only the level of stress does.

Now, suppose that ammunition lot i would be stored in depot k_1 during $[0, t_1]$, in depot k_2 during $[t_1, t_2]$ and finally transferred to depot k_3 and stored during $[t_2, t_3]$. In this case, a routing sequence of locations of depots becomes (k_1, k_2, k_3) along with associated duration $[0, t_1], [t_1, t_2], [t_2, t_3]$, respectively.

First of all, we define the predicted cumulative proportion defective of lot i in the lth sequence of storage, depot k_l, as $\hat{p}_i^{(l)}(t)$. For lot i that would have been stored in depot k_1 from the beginning to time t_1, the expected cumulative proportion defective by time t is predicted as

$$\hat{p}_i(t) = \hat{p}_i^{(1)}(t) = exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_1)} t)/(1 + exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_1)} t)) \tag{12}$$

where $0 < t < t_1$ and $\hat{\beta}_i^{(k_1)} = exp(\hat{\gamma}_{k_1})$.

Next, ammunition lot i is moved to depot k_2 from depot k_1. Depot k_2 has an equivalent starting time s_1 which would have produced the same proportion defective as in depot k_1 at time t_1 if the ammunition lot had been stored in depot k_2 from the beginning. Such s_1 would satisfy the following relationship: $\hat{p}_i^{(2)}(s_1) = \hat{p}_i^{(1)}(t_1)$ or $\hat{\beta}_0 + \hat{\beta}_i^{(k_2)} s_1 = \hat{\beta}_0 + \hat{\beta}_i^{(k_1)} t_1$. Thus,

$$s_1 = (\hat{\beta}_i^{(k_1)} / \hat{\beta}_i^{(k_2)}) t_1. \tag{13}$$

As a result, the predicted cumulative proportion defective of lot i in depot k_2 by time t after transferred from depot k_1 is

$$\hat{p}_i(t) = \hat{p}_i^{(2)}(t - t_1 + s_1) = exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_2)}(t - t_1 + s_1))/(1 + exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_2)}(t - t_1 + s_1))) \tag{14}$$

where $t_1 \leq t \leq t_2$.

Similarly, at the third sequence of the routing, depot k_3 has the equivalent starting time s_2 which satisfies $\hat{p}_i^{(3)}(s_2) = \hat{p}_i^{(2)}(t_2 - t_1 + s_1)$. Thus, s_2 is the solution of $\hat{\beta}_0 + \hat{\beta}_i^{(k_3)} s_2 = \hat{\beta}_0 + \hat{\beta}_i^{(k_2)}(t_2 - t_1 + s_1)$, i.e.,

$$s_2 = (\hat{\beta}_i^{(k_2)} / \hat{\beta}_i^{(k_3)})(t_2 - t_1 + s_1). \tag{15}$$

The cumulative proportion defective of lot i in depot k_3 by time t after transferred from depot k_1 and depot k_2 sequentially is predicted as

$$\hat{p}_i(t) = \hat{p}_i^{(3)}(t - t_2 + s_2) = exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_3)}(t - t_2 + s_2))/(1 + exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_3)}(t - t_2 + s_2))) \tag{16}$$

for time period $t_2 \leq t \leq t_3$.

In general, for lot i which is transferred from depot k_{l-1} to depot k_l at time t_{l-1}, depot k_l has the equivalent starting time s_{l-1} which satisfies the following relationship: $\hat{\beta}_0 + \hat{\beta}_i^{(k_l)} s_{l-1} = \hat{\beta}_0 + \hat{\beta}_i^{(k_{l-1})}(t_{l-1} - t_{l-2} + s_{l-2})$ where $l = 2, .., L$, $t_0 = 0$ and $s_0 = 0$. Therefore

$$s_{l-1} = (\hat{\beta}_i^{(k_{l-1})} / \hat{\beta}_i^{(k_l)})(t_{l-1} - t_{l-2} + s_{l-2}). \tag{17}$$

Consequently the cumulative proportion defective of lot i in depot k_l by time t, $t_{l-1} \leq t \leq t_l$ is estimated as follows:

$$\hat{p}_i(t) = \hat{p}_i^{(l)}(t - t_{l-1} + s_{l-1})$$

$$= exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_l)}(t - t_{l-1} + s_{l-1}))/(1 + exp(\hat{\beta}_0 + \hat{\beta}_i^{(k_l)}(t - t_{l-1} + s_{l-1}))) \tag{18}$$

for time period $t_{l-1} \leq t \leq t_l$.

Thus, the predicted cumulative proportion defective of ammunition lot i, $\hat{p}_i(t)$, for step-stress pattern (k_1, k_2, k_3) along with associated duration $[0, t_1], [t_1, t_2], [t_2, t_3]$ consists of segments of the $\hat{p}_i^{(1)}(\ )$, $\hat{p}_i^{(2)}(\ )$, and $\hat{p}_i^{(3)}(\ )$.

4. ILLUSTRATION

In order to illustrate implementation procedures for the methods suggested in this paper, a numerical example is generated based on the parameters used in the guideline of the U.S. Army Ammunition Surveillance Procedure [12].

Suppose 20 ammunition lots were used for experiment to estimate β_i's where $(i = 1, .., 20)$. The experimental lots are exposed to four different levels of constant stress representing the conditions of depot 1 (lots 1,..,5), depot 2 (lots 6,..,10), depot 3 (lots 11,..,15), and depot 4 (lots 16,..,20). All the lots are inspected annually based on sample size of 20 (n_{ij}). Inspection starts when the lot is 3 years old ant it is done thereafter every three years until 15th year. Table 1 contains information regarding the series of the number of defective items (y_{ij}) found in 20 ammunition lots (N).

This information is used to obtain ML estimates of parameters in the within-lot model (1) ($\hat{\beta}_0$, $\hat{\beta}_1$,..,$\hat{\beta}_{20}$ and $\hat{\Delta}$). For this step, PROC LOGISTIC of a statistical package SAS [8] is used. In Figure 1, sample patterns of the actual deterioration (Actual:X_{ij} / Y_{ij}) are overlaid to those of the estimated deterioration (Fitted: $(exp(\hat{\beta}_0 + \hat{\beta}_i t_{ij})/(1 + exp(\hat{\beta}_0 + \hat{\beta}_i t_{ij})))$ against time t_{ij}).

The between-lot model (2) is formed to relate deterioration rates to the depot characteristics. $ln\hat{\beta}_i$ is used as dependent variable and the four dummy variables (z_{i1},..,z_{i4}) representing 4 different depots are used as covariates without an intercept: $z_{i1} = 1$ for ammunition lots stored in depot 1, otherwise $z_{i1} = 0$; $z_{i2} = 1$ for ammunition lots stored in depot 2, otherwise $z_{i2} = 0$; $z_{i3} = 1$ for ammunition lots stored in depot 3, otherwise $z_{i3} = 0$; and $z_{i4} = 1$ for ammunition lots stored in depot 4, otherwise $z_{i4} = 0$. Given such $(z_{i1}, .., z_{i4})$ as well as ($\hat{\beta}'_i$, and $\hat{\Delta}$), the IMSL subroutine ZSPOW [4], is applied to (6,7) in order to obtain $\hat{\gamma}_1, .., \hat{\gamma}_4$ and $(Z'\hat{W}Z)^{-1}$. These estimates are summarized in Table 2.

Now we apply these results to predict the proportion defective of an ammunition lot i which would be transferred from depot 1 to depot 4

following the route given in Table 3. The route given in Table 3 can be related to the following: ammunition lot i would be stored in the temporary depot 1 for 1 year and transferred to the permanent depot 2 where it would be kept for the next 4 years. The lot i then would be sent to the depot 3 in warship and stored for 1 year and brought back to depot 4 where it would remain until ultimate usage.

The expected proportion defective in depot 1 by time t, $0 \leq t \leq 1$, can be predicted as

$$\hat{p}_i(t) = \hat{p}_i^{(1)}(t) = \frac{exp(-5.8551 + exp(-1.6058)t)}{(1 + exp(-5.8551 + exp(-1.6058)t))}. \tag{19}$$

The expected proportion defective of ammunition lot i stored in depot 2 during $1 \leq t \leq 5$ after being transferred from depot 1 is predicted as

$$\hat{p}_i(t) = \hat{p}_i^{(2)}(t - 0.0606) = \frac{exp(-5.8551 + exp(-1.5432)(t - 0.0606))}{(1 + exp(-5.8551 + exp(-1.5432)(t - 0.0606)))}. \tag{20}$$

Next, the expected proportion defective of ammunition lot i stored in depot 3 during $5 \leq t \leq 6$ after depot 1 and depot 2 is predicted as

$$\hat{p}_i(t) = \hat{p}_i^{(3)}(t - 0.9016) = \frac{exp(-5.8551 + exp(-1.3566)(t - 0.9016))}{(1 + exp(-5.8551 + exp(-1.3566)(t - 0.9016)))}. \tag{21}$$

Similarly, the expected proportion defective of ammunition lot i stored in depot 4 during $t \geq 6$ after storage in depot 1, 2 and 3 is predicted as

$$\hat{p}_i(t) = \hat{p}_i^{(4)}(t - 2.5887) = \frac{exp(-5.8551 + exp(-0.9547)(t - 2.5887))}{(1 + exp(-5.8551 + exp(-0.9547)(t - 2.5887)))}. \tag{22}$$

Figure 2 shows the segments of the estimated cumulative proportion defectives obtained under the four different depot conditions. First, the

quality of the ammunition lot deteriorates according to the average condition of depot 1 up to time t_1. After it is transferred to depot 2, the stockpile deteriorates following the pattern fitted for depot 2, starting at the accumulated proportion defective due to the condition of depot 1. Similarly, when the lot is shipped from depot 2 to depot 3, from depot 3 to depot 4, deteriorating patterns follow the corresponding logistic models fitted under each depot condition, respectively starting at the accumulated proportion defective at the previous depots. Figure 3 gives their connection representing the predicted cumulative proportion defective of the ammunition lot under the step-stress described in Table 3.

Under the route given in Table 3, the proportion defectives of ammunition lot i in depot 4 by $t = 10$ is predicted as follows:

$$\hat{p}_i(10) = \hat{p}_i^{(4)}(7.4113) = 0.0473. \tag{23}$$

Similar calculation provides $\hat{p}_i(12) = 0.0969$, $\hat{p}_i(13) = 0.1365$, and $\hat{p}_i(14) = 0.1880$.

When the substandard quality of the ammunition lot is set at $p = 0.15$, this ammunition lot should be used or replaced, at the latest, by the end of 13th year of storage in depot 4 provided it had followed the route given in Table 3.

5. DISCUSSION

A two-stage random effect logistic regression analysis is applied to predict the declining quality of ammunition stockpile when a series of location and the duration of storage associated with the ammunition lot is given. First of all, average deterioration rates are estimated using constant stress model. Next the prediction method for the cumulative proportion defective of an ammunition lot is described using a step-stress model.

We use a logistic regression model to fit deteriorating patterns of ammunition stockpile. The patterns of declining qualities are observed based on sampling inspection. Other alternatives to the logistic model include complementary log-log model and probit model (McCullagh [6]). For instance, the complementary log-log and the probit within-lot models which correspond to the logistic within-lot model (1) would be $p_{ij} = 1 - exp(-exp(\beta_0 + \beta_i t_{ij}))$, and $p_{ij} = \Phi(\beta_0 + \beta_i t_{ij})$, respectively, where $\Phi(.)$ is the standard normal integral. Even in these model specifications, parameters β_i's represent the deterioration rates as described in the logistic regression model (1). To estimate β_i's, the maximum likelihood estimation methods given in (3) can be applied. The corresponding between-lot model analyses are essentially the same as described in the two stage estimation. Choice of the specific within-lot model among logistic, probit and complementary log-log models depends on the pattern of data gathered from the experiment.

When the response is taken in terms of the change in a certain attribute of an item such as water content in propellant, rather than counting the number of defectives based on a certain sampling scheme, a nonlinear model can be used to formulate deteriorating patterns of ammunition lots. A typical example is use of a negative exponential growth curve model. How-

ever, the selection of appropriate nonlinear model again depends on the observed declining characteristic of the quality of ammunition stockpile.

In this paper, different levels of depot conditions are viewed step-stress. Degradation is assumed to be independent of the sequence of step-stress, (i.e., sequence of depot routing). Sometimes abrupt changes between two consecutive depots may cause faster deterioration of stockpile than expected in these two depots. Developing models that accommodate sequential dependence is left as one of areas for further study.

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Table 1: Number of Defectives y_{ij} Observed in Sample Size of 20 at the jth Inspection of Lot i

	group 1					group 2				
	lot i									
t_{ij}	1	2	3	4	5	6	7	8	9	10
3	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0
12	0	1	1	0	0	1	0	1	1	1
15	1	1	1	1	1	1	1	1	1	1
$\hat{\beta}_i$	0.1494	0.1966	0.1966	0.1494	0.1494	0.1966	0.1494	0.1966	0.1966	0.1966
$se(\hat{\beta}_i)$	0.0813	0.0644	0.0644	0.0813	0.0813	0.0644	0.0813	0.0644	0.0644	0.0644

	group 3					group 4				
	lot i									
t_{ij}	11	12	13	14	15	16	17	18	19	20
3	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	1	1	1	1
9	1	0	0	0	2	1	2	2	3	2
12	1	1	1	1	3	3	4	4	5	4
15	1	1	1	1	5	5	6	6	8	6
$\hat{\beta}_i$	0.2197	0.1966	0.1966	0.1966	0.2836	0.3827	0.3520	0.3520	0.3827	0.3520
$se(\hat{\beta}_i)$	0.0583	0.0644	0.0644	0.0644	0.0471	0.0414	0.0421	0.0421	0.0414	0.0421

Table 2: Fitted Between-Lot Model

l	$\hat{\gamma}_l$	$Z'\hat{W}Z$			
1	-1.6058	0.0631	0.0316	0.0257	0.0163
2	-1.5432	0.0316	0.0490	0.0236	0.0149
3	-1.3566	0.0257	0.0236	0.0297	0.0121
4	-0.9547	0.0163	0.0149	0.0121	0.0106

Table 3: Scenario for Location and Duration of Storage

Depot	1	2	3	4
Period	[0,1]	[1,5]	[5,6]	[6,−]

Figure 1: Deteriorating Patterns of Ammunition Quality

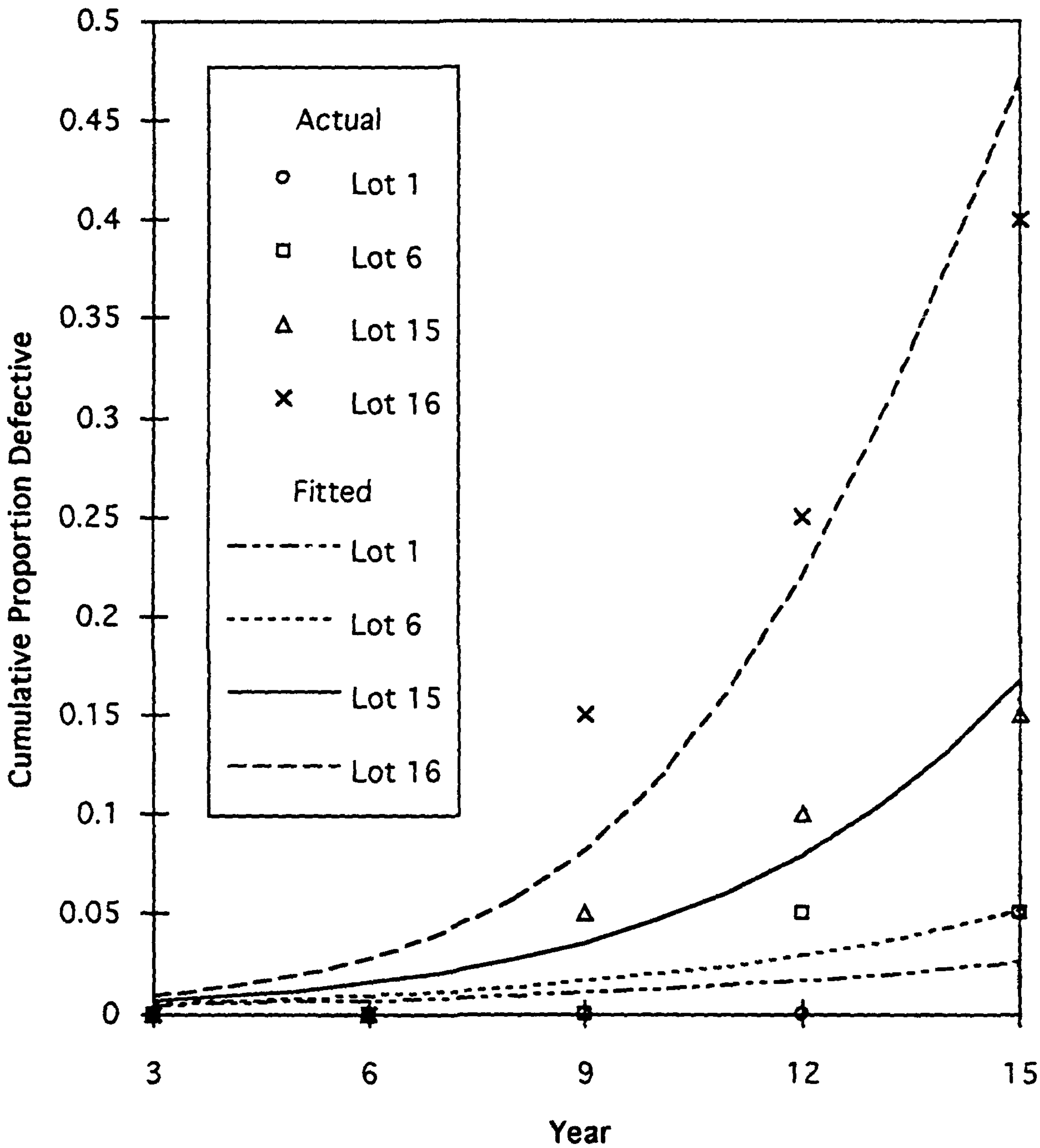


Figure 2. Segments of the Predicted Cumulative Proportion Defective of the Ammunition Lot under Constant Stress

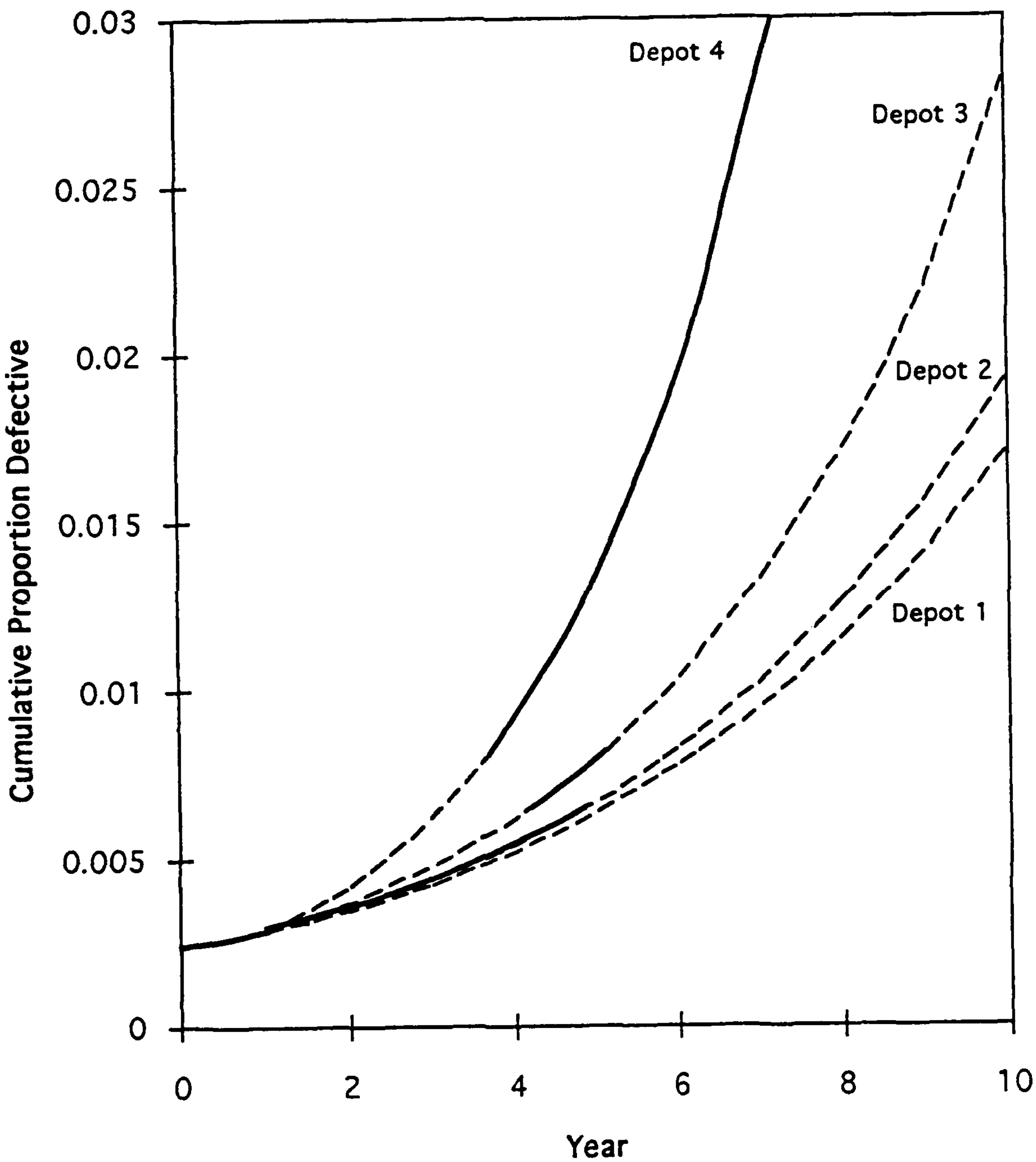
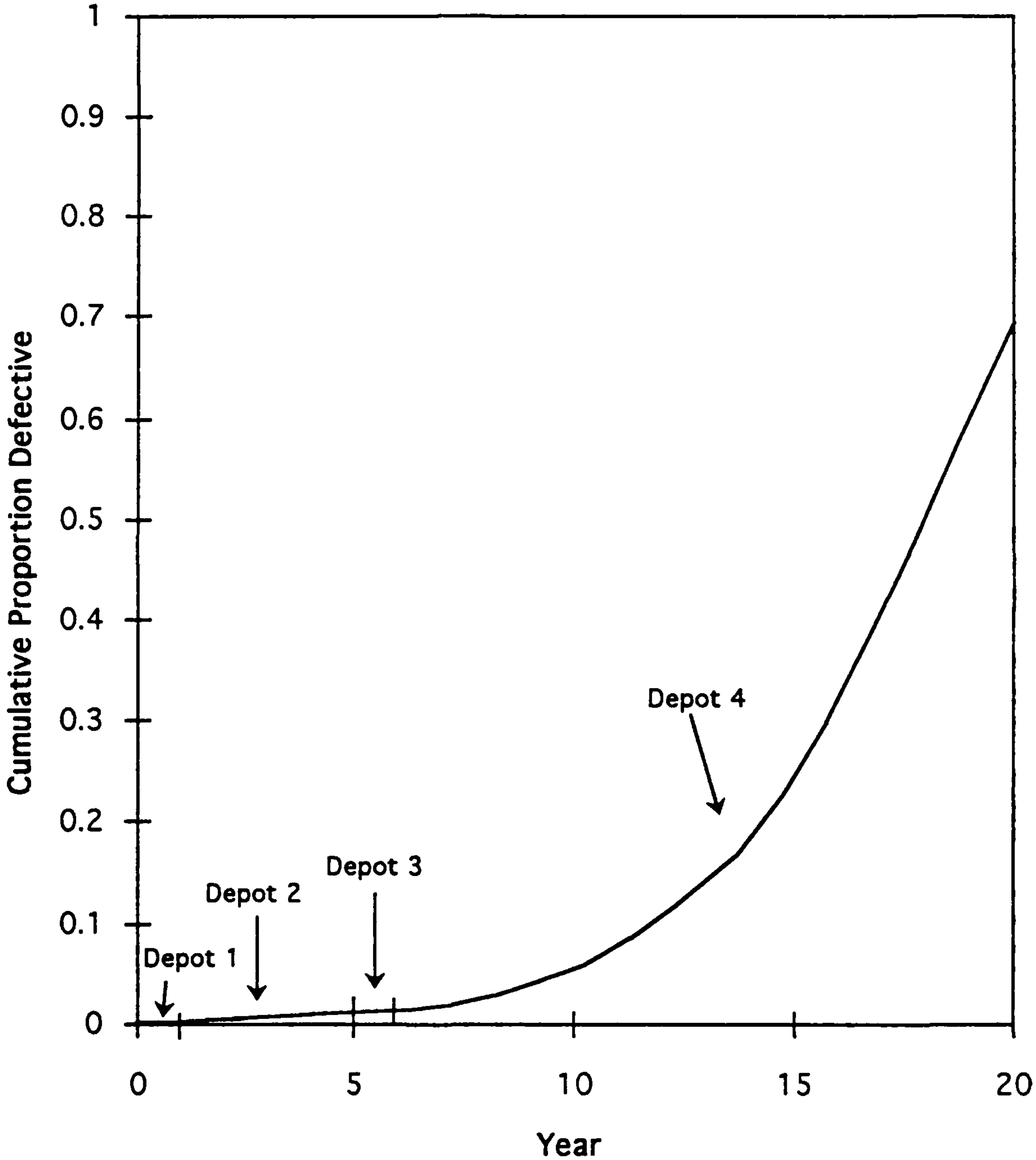


Figure 3. Predicted Cumulative Proportion Defective of the Ammunition Lot under Step-Stress



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